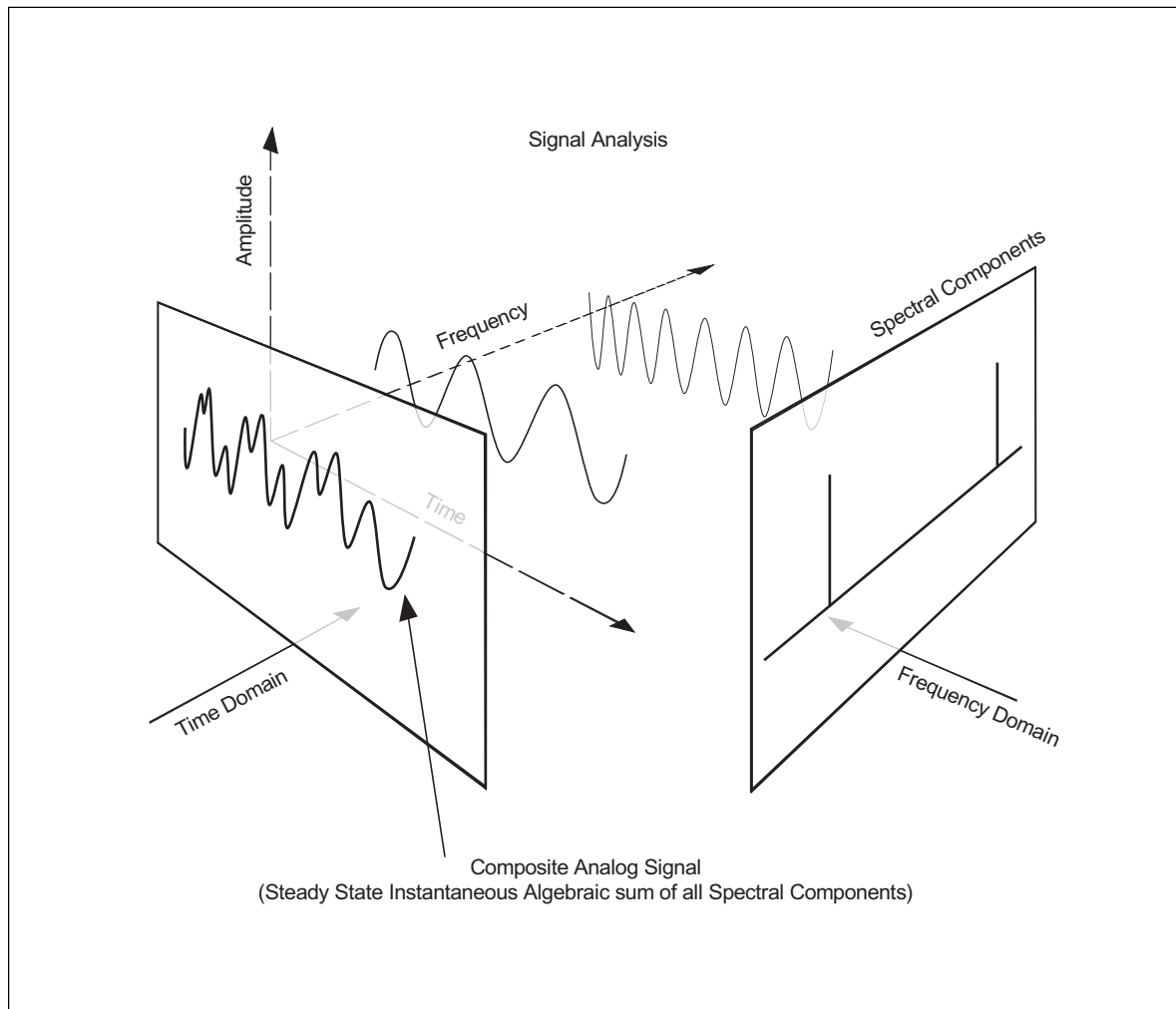**D▶ DATEL** ®

INNOVATION and EXCELLENCE

Understanding Data Converters' Frequency Domain Specifications

Innovation and Excellence in Precision Data Acquisition

By **Bob Leonard**, *DATEL, Inc.*

TIME DOMAIN VS. FREQUENCY DOMAIN

Time domain data collected from an A/D converter is mapped into the frequency domain using the Fast Fourier Transform (FFT) algorithm.

DATEL was founded in 1970 as a producer of high performance Analog-to-Digital Converters and Data Acquisition products. Our administrative offices, engineering, modular and subsystem production facilities and hybrid production facilities qualified to MIL-STD-1772, are housed in our 180,000 square foot facility in Mansfield, Massachusetts.

Panel Products

Digital Panel Meters
Process Monitors
Printers
Calibrators

Data Acquisition Boards

VME
Multibus
PC/AT

Data Conversion Components

A/Ds
Amplifiers
Sampling A/Ds
Filters
Sample-Holds
HDASs
D/As
Multiplexers

Power Products

DC-DC Converters
AC-DC Converters

Fourier Transform for A/D's

The Fourier Transform is intended to operate on continuous "waveform" data from $-\infty$ to $+\infty$.

$$X(f) = \int_{-\infty}^{\infty} x(t)\ e^{-j\omega t}dt$$

Where: $\omega = 2\pi f$

Discrete Fourier Transform (DFT) is used on sampled A/D data. The ideal continuous waveform from $-\infty$ to $+\infty$ has been replaced with *sampled* points on a waveform for a limited time period.

$$X_D(n\Delta f)= \sum_{K=0}^{N-1} X(k\Delta t)e^{-j2\pi n\Delta fk\Delta t}\Delta t$$

Where:　N = Total # of points in the record

　　　$n\Delta f$ = Finite # of frequency points

　　　Δt = Sampling interval

　　　k = Integer

The Fast Fourier Transform (FFT) Algorithm is used in implementing the Discrete Fourier Transform due to the FFT'S mathematical efficiency.

The Fast Fourier Transform is a mathematically efficient algorithm to supplant the Discrete Fourier Transform. Likewise the DFT is used to supplant the Continuous Fourier Transform for time sampled data. In principle, most aspects of the CFT transfer to the DFT and FFT, however there are nuances that demand attention.

N data points in the time domain produce N/2 points (amplitude and phase) in the frequency domain.

FFT VERSUS DFT PROCESSING

# OF POINTS	FFT IMPROVEMENT
N	$N^2/N\ LOG_2\ N$
128	×18.2
256	×32
512	×56.9
1024	×102.4
2048	×186.2
4096	×341.3
8192	×630.2

The Fast Fourier Transform (FFT) requires NLog2 N operations (multiplication & addition) while the Discrete Fourier Transform (DFT) requires N2 operations.

Fast Fourier Transform

Weaknesses/Cures

Aliasing/Nyquist Sampling

Leakage/Windowing

Picket-Fence effect /# of FFT Points

Successful application of the FFT demands an appreciation of three basic limitations; aliasing, leakage and picket-fence effect.

Observation of the Nyquist Sampling rate, utilizing windows to weight the non-infinite sequenced data and choosing an appropriate number of FFT points provide the appropriate solutions to these limitations.
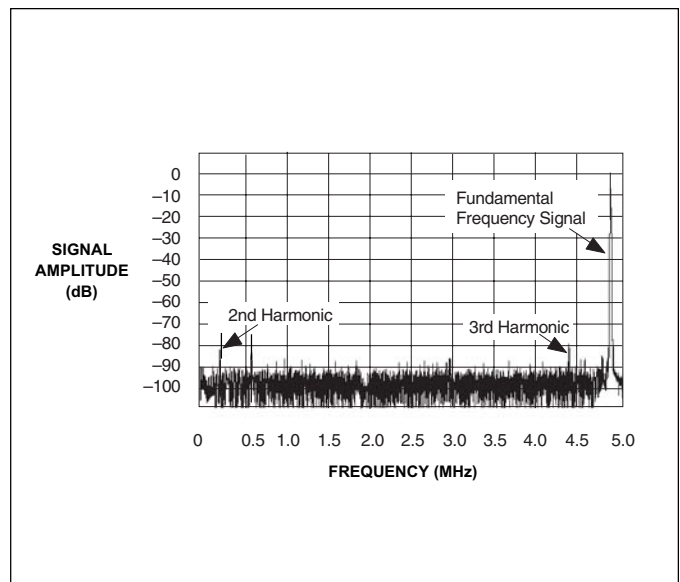

A non-ideal FFT plot of an actual Sampling A/D illustrates the input frequency, harmonics generated and the noise floor. A single-tone frequency of 4.85 MHz was the input for this 12-bit, 10 MHz device.

ALIAS FREQUENCY CAUSED BY INADEQUATE SAMPLING RATE
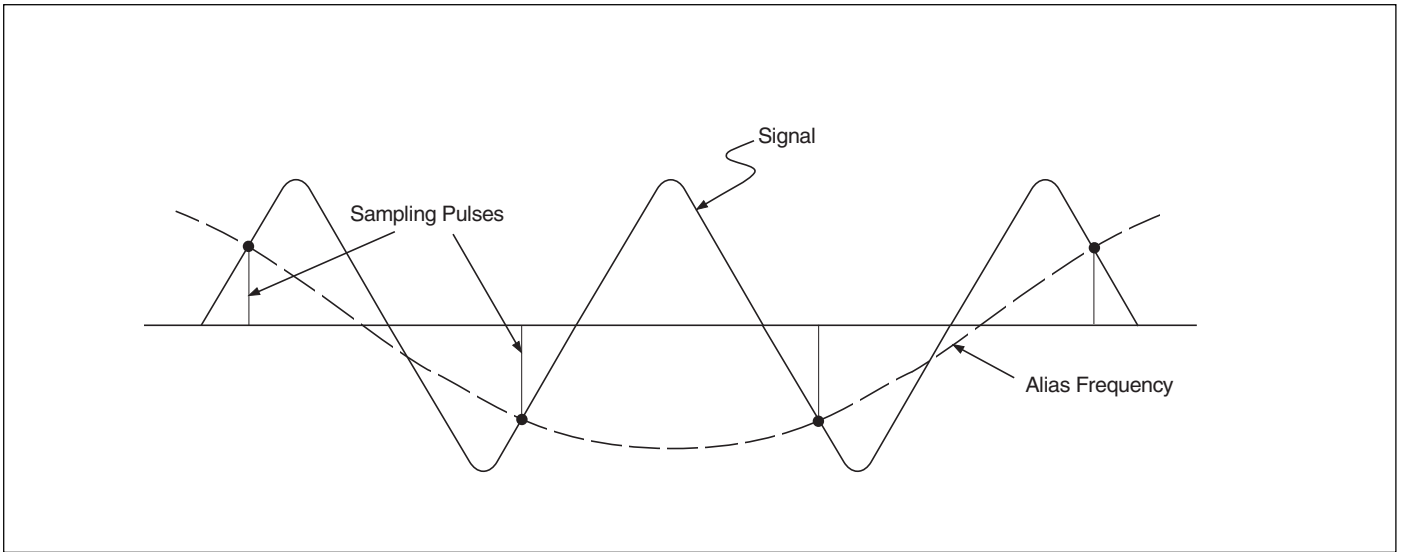
An inadequate sampling rate has the effect of producing an alias frequency in the recovered signal. Sampling at a rate less than twice per cycle results in an alias which is significantly different from the original frequency.

FREQUENCY SPECTRA DEMONSTRATING THE SAMPLING THEORUM

DATEL
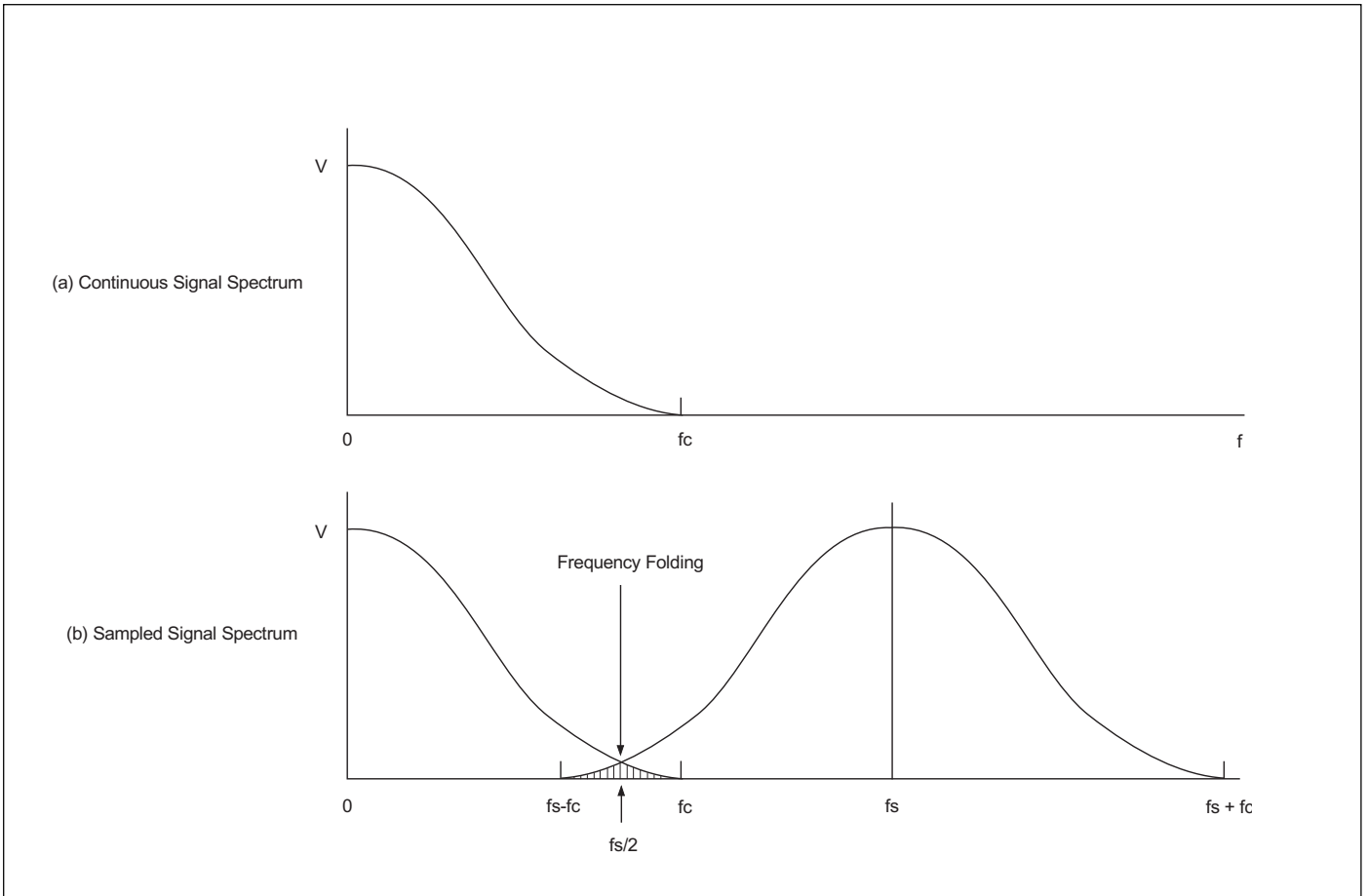
Frequency Domain Specifications

Signal-to-Noise Ratio & Distortion (Sinad)

Signal-to-Noise Ratio without Distortion

Total Harmonic Distortion

In-Band Harmonics

Spurious Free Dynamic Range (SFDR)

two-Tone Intermodulation Distortion

Noise Power Ratio (NPR)

Effective Bits

Some key Frequency Domain specifications for Sampling A/D converters are listed. Understanding how these are defined and under what conditions is as important as knowing the FFT pitfalls and cures.

Frequency Spectra

Demonstrating The Sampling Therorum

The Nyquist Sampling Theorum requires that a continuous bandwidth-limited analog signal, with frequency components out to *fc,* must be sampled at a rate fs which is a minimum of *2fc.*

If the sampling frequency fs is not high enough, part of the spectrum centered about fs will fold over into the original signal spectrum (frequency folding). Frequency folding can be eliminated in two ways: first by using a high enough sampling rate and second by filtering the signal before sampling to limit its bandwidth to fs/2.

Frequency Folding and Aliasing

Procedure:

Analyze the input frequency as: Fin = K(Fs/2) + ΔF

Where: Fin = Input Frequency

Fs = Sampling Rate

K = Odd or even integer

(multiple of half the sampling rate)

(need to determine K when substituting into the formula)

ΔF = Differential change in frequency needed to equate the formula

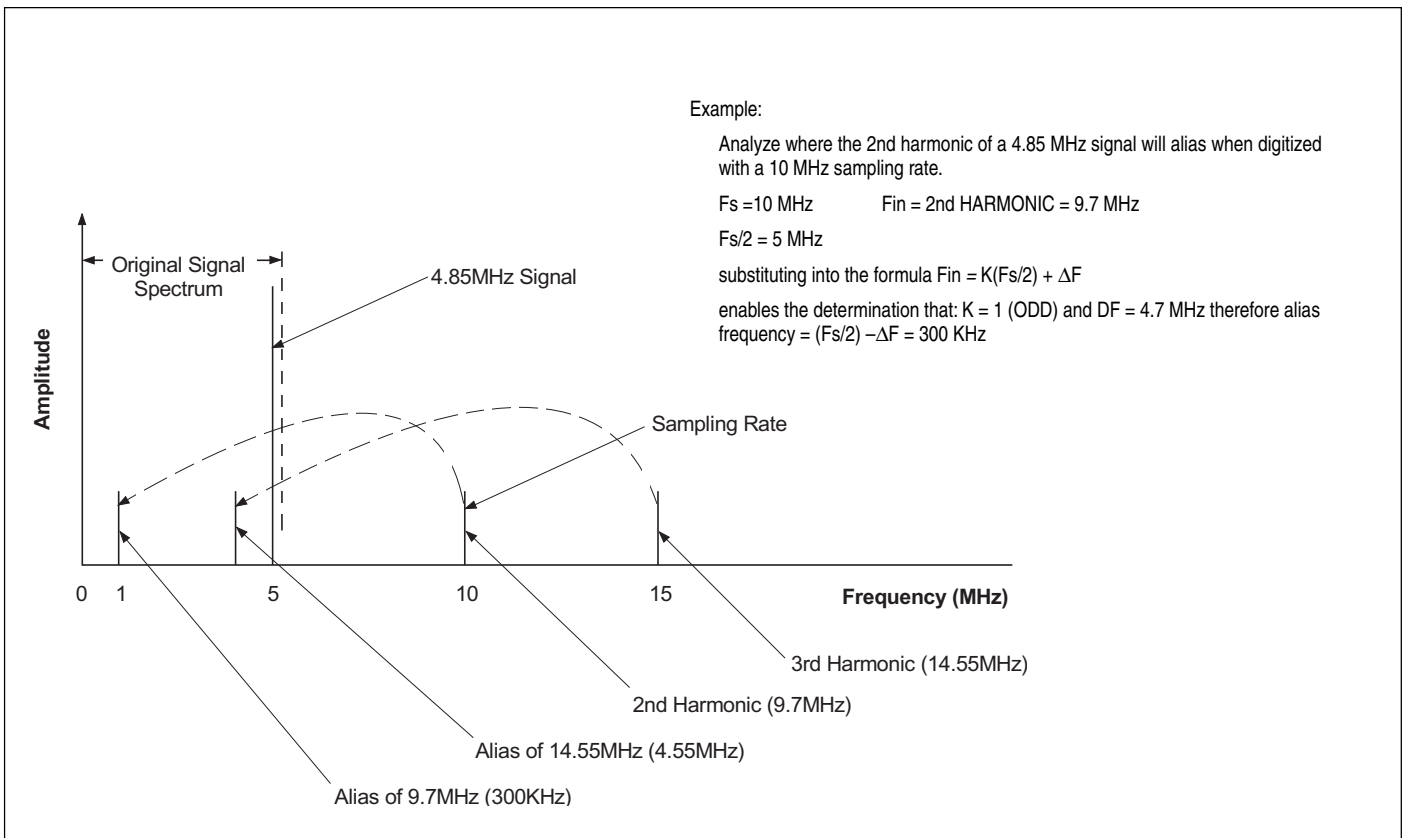(need to determine ΔF when substituting into the formula)

If K is odd: Alias Frequency = (Fs/2) – ΔF

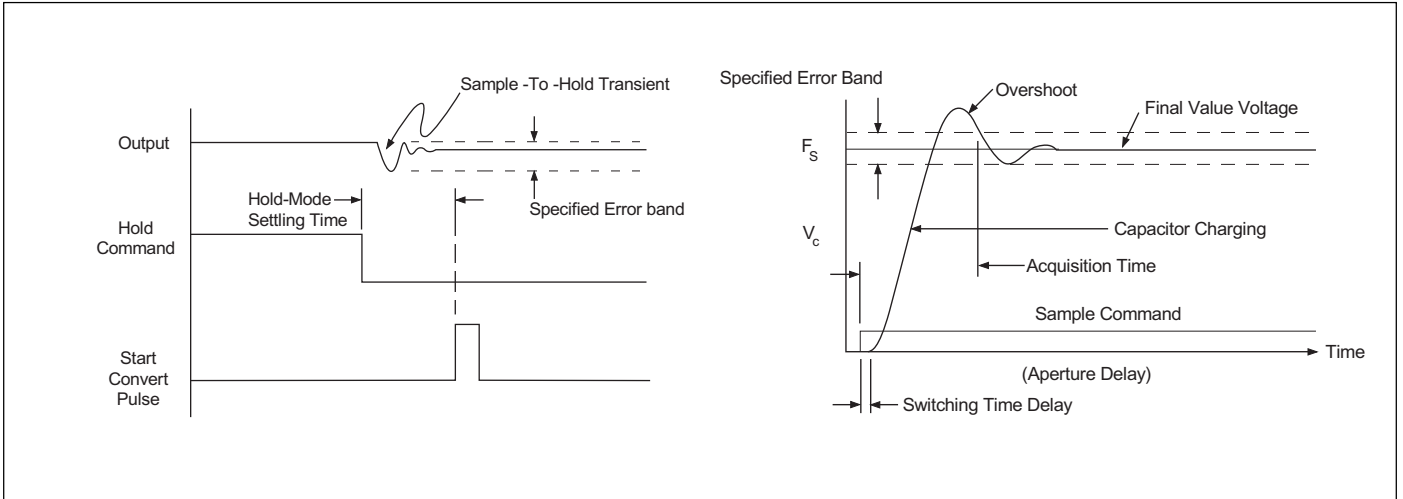If K is even: Alias Frequency = ΔF

The aliasing formulas are useful in determining where a harmonic will alias back into the signal spectrum. Conversely, a harmonic or spurious frequency can suggest possible frequencies that caused them. An example could be a system clock operating at a much higher frequency appearing as an alias in the signal spectrum.

An initial disconcertment over two unknowns and one formula is reduced once familiarity with the substitution process is practiced.

FREQUENCY FOLDING AND ALIASING

Example:

Analyze where the 2nd harmonic of a 4.85 MHz signal will alias when digitized with a 10 MHz sampling rate.

Fs =10 MHz   Fin = 2nd HARMONIC = 9.7 MHz

Fs/2 = 5 MHz

substituting into the formula Fin = K(Fs/2) + ΔF

enables the determination that: K = 1 (ODD) and DF = 4.7 MHz therefore alias frequency = (Fs/2) –ΔF = 300 KHz

Original Signal Spectrum

4.85MHz Signal

Amplitude

Sampling Rate

0 1   5   10   15   **Frequency (MHz)**

3rd Harmonic (14.55MHz)

2nd Harmonic (9.7MHz)

Alias of 14.55MHz (4.55MHz)

Alias of 9.7MHz (300KHz)

KEY SAMPLE-HOLD SPECIFICATIONS

Frequency Folding and Aliasing

Utilizing the alias formulas, a 2nd harmonic (9.7 MHz) of a 4.85 MHz signal frequency when sampled at 10 MHz will appear as an alias frequency of 300 KHz on an FFT plot. Similarly, a 3rd harmonic of the 4.85 MHz signal (14.55 MHz) would then yield K = 2 (even) and DF = 4.55 MHz. The alias frequency would appear as DF, or 4.55 MHz, on the FFT plot.

APERTURE UNCERTAINTY
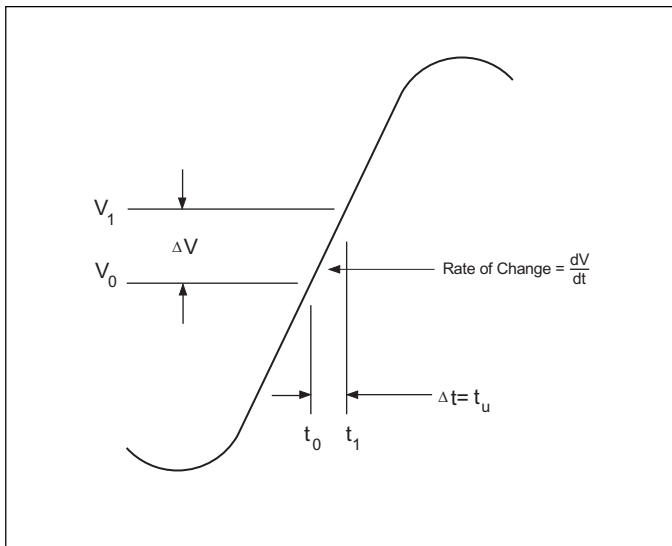
Aperture Uncertainty

The actual voltage digitized by the data converter depends on the input signal slew rate and the "aperture time." The "aperture time" is application and architecture dependent.

The "aperture time," Dt, could be the conversion time of the A/D converter used without a Sample-Hold. The "aperture time" could also be the aperture delay time specification of a Sample-Hold. Applications with repetitive sampling, such as those utilizing the FFT, can use the aperture uncertainty (tu) specification. The aperture delay is just phase information which is not required in identifying the frequency components.

Key Sample -Hold Specifications

Among the key specifications for Sample-Hold are Acquisition Time and Hold Mode Settling Time.

Acquisition Time of a S/H starts with the sample command and end when the voltage on the hold capacitor enters and stays in the error band. Acquisition time is defined for a full-scale voltage change, measured at the hold capacitor.

When an A/D converter follows a S/H, the start conversion pulse must be delayed until the output of the S/H has had enough time to settle within the error band and stays there.

"CLASSIC" SAMPLE-HOLD
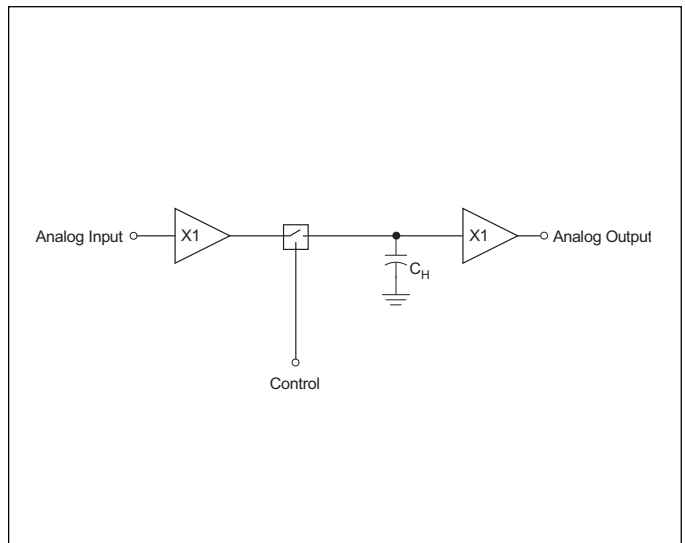
"Classic " Sample-Hold

The sampling process begins for many applications with the Sample-Hold (S/H) in front of the Analog-to-Digital Converter

The "classic" open loop follower sample-hold architecture has a buffer in front of the switch to quicken capacitor charging and gives the S/H a high input impedance. Adding a buffer behind the hold capacitor reduces capacitor charge bleeding and output droop.

Determining Maximum Frequency

Sinusoidal Inputs

Given:	E = [dv/dt]T
Where:	E = Voltage error tolerable
	dv/dt = Signal slew rate
	T = "Aperture time"
Sinusoidal signal form:	$V_{in} = V \cdot SIN(2\pi ft)$
Taking the 1st derivative:	$dV/dt = (2\pi f)V \, COS(2\pi ft)$
At zero crossing:	$t = n/2f$ (n = 1,2,3...)
Yielding:	$dV = (2\pi fV)dt$

Maximum frequency (f) = $\dfrac{dV}{dt(2\pi)V}$

   The Sample-Hold ideally stores an instantaneous voltage (sample value) at a desired instant of time. The constraint on this time is aperture uncertainty for sinusoidal waveforms.
   Noise affecting the logic threshold level of the sample-hold command creates a timing uncertainty in the switch and driver circuit (aperture uncertainty). The resulting error voltage for a given period of time increases with the rate of change of the input signal.
   Determining the maximum possible frequency to digitize a ±10V sinusoid to ±1/2 LSB accuracy to 12-bits yields (for aperture uncertainty = 50ps):

$$f = \frac{2.44mV}{(50ps)2\pi 10V} = 776 \text{ KHz}$$

Logarithms

Some common algebraic manipulations of logarithms are shown as many of the frequency domain specifications will be expressed in dBs. One useful rule(of thumb is 6 dB of dynamic range per bit of resolution.

The test condition of –0.5dB below Full Scale is becoming important in Frequency Domain specifications and testing. The IEEE has recommended –0.5dB as one of the test conditions for the frequency domain specifications for Waveform Instrumentation.

dB = 20 log(V2/V1)
FOR V2N1 = 10,000 >>> 20 log(10,000) = 80dB
FOR V2/V1 = 0.0001 >>> 20 log(0.0001) = –80dB
FOR A 12-BIT CONVERTER, 212 = 4096
20log(4096) = 72.2dB >>> 72.2dB/12-BITS = 6.02dB/Bit

$-0.5dB = 10^{(-0.5/20)} = 0.944$
$-3dB = 10^{(-3/20)} = 0.707$
$-6dB = 10^{(-6/20)} = 0.5$
$-20dB = 10^{(-20/20)} = 0.1$

Frequency Domain Specifications

Some key Frequency Domain specifications for Sampling A/D converters are listed. Understanding how these are defined and under what conditions is as important as knowing the FFT pitfalls and cures

Signal-to-noise ratio & distortion (SINAD)
   Signal-to-noise ratio without distortion
   Total harmonic distortion
   In-band harmonics
Spurious free dynamic range (SFDR)
   Two-tone intermodulation distortion
   Noise power ratio (NPR)
   Effective bits

Signal-to-noise ratio (SNR)

Signal-To-Noise Ratio is the ratio of the RMS signal to the RMS noise. The RMS noise includes all nonfundamental spectral components which in a non-ideal A/D would include the harmonics, any spurious frequencies and the noise floor of the device, but excluding dc.

Higher resolution A/Ds reduce the "quantization noise" further, enabling a better signal-to-noise ratio to be obtained.

For an ideal A/D:

$$SNR = \frac{RMS \text{ signal}}{RMS \text{ noise}} = 6.02n + 1.76dB$$

Where n = 0 OF BITS

Resolution	Ideal SNR
8 Bits	49.9 dB
10 Bits	62.0 dB
12 Bits	74.0 dB
14 Bits	86.0 dB
16 Bits	98.1 dB

Signal-to-noise ratio & distortion (SINAD)

The "classic " definition of SNR included the harmonics

   The signal is the fundamental frequency

   The noise is anything unwanted which included the harmoics, any spurious frequencies and noise floor (offset error excluded).

   The "classic" definition of SNR is now often stated as:

   Signal-to-noise ratio & distortion (SINAD)

   If signal-to-noise ratio is specified without distortion, SINAD can be determined as follows:

SNR & Distortion = $-20 \text{ Log} \sqrt{(10^{-SNR \text{ W/O DIST}/10}) + (10^{+THD/10})}$

Traditionally, when a Signal-to-Noise Ratio specification was published, the signal was the fundamental frequency and the noise was anything unwanted (harmonics, spurious frequencies, noise floor).

Today, it is important to determine if the SNR specified is with or without distortion. If the SNR w/o distortion and the Total Harmonic Distortion (THD) are specified, SINAD can be calculated.

In Data Converters, the published SINAD specification may suggest upon calculation that better SNR w/o Distortion and Harmonic Distortion specifications than published exist.

The more conservative individual guaranteed specifications are sometimes due to the subtle A/D design trade-offs. If the harmonics are lower, the noise floor may be higher, however the guaranteed SINAD specification can still be maintained.

RMS Signal

Given the sinusoidal expression: $v(t) = A\sin\omega t$

Amplitude A = 1/2 the Full Scale Range (FSR)

$$\text{RMS signal} = \frac{A}{\sqrt{2}} = \frac{FSR/2}{\sqrt{2}} = \frac{(2^{n-1})q}{\sqrt{2}}$$

Where q = LSB Size

As the theoretical calculations for Signal-to-Noise Ratio are developed, some algebraic manipulations allow the formula to be developed as a function of the # of bits of the data converter.

RMS Noise

Quantization error = ± 1/2 LSB

RMS Quantization noise = $Qn = q\sqrt{12}$

WHERE q = LSB SIZE

To get equal distribution of the quantization error, a triangle waveform could be assumed for the quantization error with a period T utilized to derive the RMS quantization noise after some integration as follows:

$$Q_n^2(t)\Big|_{rms} = \frac{1}{T}\int_{-T/2}^{T/2}\left(\frac{qt}{T}\right)^2 dt = \frac{q^2}{T^3}\left(\frac{t^3}{3}\right)_{-T/2}^{T/2}$$

$$Q_n^2(t)\Big|_{rms} = \frac{q^2}{3t^3}\left[\left(\frac{T}{2}\right)^3 - \left(\frac{-T}{2}\right)^3\right] = \frac{q^2}{12}$$

$$Q_n^2(t)\Big|_{rms} = q/\sqrt{12}, \text{ Yielding:}$$

$$\text{SNR} = \frac{V_{SIGNAL}(RMS)}{V_{NOISE}(RMS)} = \frac{q(2^{n-1})/\sqrt{2}}{q/\sqrt{12}} = 2^{n-1}\sqrt{6}$$

$$\text{SNR} = 20\text{ Log}\left[\sqrt{6}\,(2^{n-1})\right] = 6.02n + 1.76\text{dB}$$

An ideal A/D is limited by the inherent quantization error of ±1/2 LSB maximum where an LSB = $FSR/2^n$ (n = # of bits, FSR = Full Scale Range).

The quantization error is similar to the analog input being in the presence of white noise and is often expressed as quantization noise. The root-mean-square value of the quantization error is used as the average quantization error is zero (just as likely to be +1/2 LSB as –1/2 LSB of error).

Differential nonlinearities in data converters which are not able to meet the ideal quantization errors contribute to degraded Signal-to-Noise ratio.

Spectral Averaging

A spectral average of the FFT allows distinguishing the harmonics and the noise floor level much easier. Now the magnitudes at each FFT point can be averaged to help overcome the randomness of the noise.

The quantization noise creates a grassy pattern at the bottom of the FFT plot known as the noise floor. An ideal 12-bit A/D with a full scale sinewave input, a 4096 point FFT and a Blackman-Harris window (ENBW = 2) yields a theoretical noise floor of 105.1 dB utilizing the formula shown.

The noise in a data converter can be attributed to the quantization noise (number of bits), the inherent converter noise (semiconductor junction noise, resistor noise) and a frequency dependent term (the noise due to aperture uncertainty).
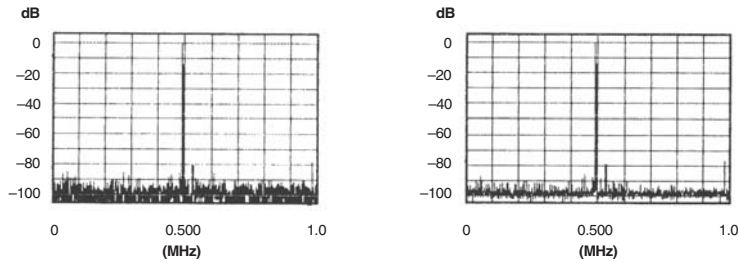
Signal-to-noise ratio cautions

Determine if SNR & distortion (SINAD) or SNR without distortion is being used.

Make sure full scale signals are being tested.

1) –0.5dB down from full scale is conservative test condition.

2) Be aware if full scale is not specified at all
   Make sure the signal frequency is specified.

1) Characterization to nyquist is conservative test condition. Additional concerns could include:

1) How many FFT points were used.

2) What windowing function was used.

SPECTRAL AVERAGING

$$\text{Average Noise Floor} = 6.02n + 10 \log \left[\frac{3N}{\pi \times (\text{ENBW})} \right] \text{dB}$$

Practically

$$\text{NOISE}_{total} \text{ (rms)} = \sqrt{[\text{NOISEQ (rms)}]2 + [\text{NOISEC (RMS)}]2 + [\text{NOISEA (rms)}]2}$$

Where: n = # of bits resolution
N = Number of data points
ENBW = Equivalent Noise
Bandwidth of the Window Function

Signal-to-noise ratio cautions

Comparison of various Data Converter Signal-to-Noise Ratio specifications requires similar test conditions.

Full Scale signals (–0.5dB down or about 94% of the Full Scale Range) characterized through Nyquist are recommended. A smaller full scale range may not exercise the data converter in a region where there are nonlinearities (harmonics in the frequency domain). Also, noise from such sources as aperture uncertainty may increase as a function of the input frequency. Typically, a good data converter's SNR degrades as the input signal decreases.

Further insight could include the frequency resolution of the FFT (how many FFT points) and the windowing functions utilized.

Total Harmonic Distortion (THD)

$$\text{THD}_{(RMS)} = 20 \log \sqrt{\left[10^{(\text{2nd HAR}/20)}\right]^2 + \left[10^{(\text{3rd HAR}/20)}\right]^2 + \dots}$$

Total Harmonic Distortion includes all of the harmonics by definition

Total Harmonic Distortion (THD) is the ratio of the rms sum of the harmonics to the rms of the fundamental signal. The harmonics appear at integer multiples of the fundamental frequency.

Integral Nonlinearities appear as harmonics in the frequency domain The amount of Total Harmonic Distortion is related to how many codes have an integral nonlinearity which is non-ideal.

Typically, a good data converter's THD improves as the input signal decreases.

Practically speaking, the first five harmonics are the major contributors to THD. Test conditions of various manufacturers may state the first 6, 9 or 40 harmonics, etc are used in the calculation.

Spurious Free Dynamic Range (SFDR)

In-Band Harmonics

Spurious Free Dynamic Range (SFDR) and In-Band Harmonics are similar in there definitions as the worst case Harmonic, Spurious Signal or Noise Component relative to input level. Practically speaking, this usually ends up as being the worst harmonic.(2nd Harmonic).

These are discussed here as some manufacturers use the Spurious Free Dynamic Range specification and some use the In-Band-Harmonics. These specifications differ from Total Harmonic Distortion (THD) which includes all Harmonics.

The Spurious Free Dynamic Range (SFDR) and In-Band Harmonic specifications are intended to indicate the largest harmonic, spurious frequency or noise component. This is usually the 2nd harmonic (rarely the third), but should the harmonics be indistinguishable from the noise, then these specifications cover this situation also.

These specifications indicate the usable dynamic range to the user, the range in which frequency components other than the fundamental do not exist.

Frequency Domain Test Conditions

Full Scale Input Considerations

The 12-bit, 1 MHz Sampling A/D (ADS-112) is shown for different input voltages with an input frequency, Fin of 490 KHz. Note the noise floor and harmonic changes which occur at the different full scale inputs.

In comparing A/D converter specifications, it is important that comparable test conditions were employed. The conservative, –0.5 dB down from full scale, test condition is recommended.

Frequency Considerations

The FFT plot shows an input frequency spectrum up to 500 KHz for the 12-bit, 1 MHz ADS-112. It is important in comparing specifications to make sure the input frequency is defined. Note the better specifications when the frequency is closer to DC versus close to the maximum for Nyquist sampling.

The ADS-112's FFT plot shows spectral components up to half the sampling rate (Fs). Make sure the sampling rate is defined for any FFT plots which do not use frequency for the X-axis. Some FFT plots have used "frequency bins" (to be discussed later) which would not allow the user to determine the sampling rate or input frequency (do not assume the maximum sampling rate, Fs, was used).

Effective Bits

Errors from Differential and Integral Nonlinearities, aperture uncertainty, noise and quantization error (noise) are involved in determining the # of effective bits.

The effective bit specification is a figure of merit on the overall A/D transfer function. As monotonicity is to Digital-to-Analog Converters and as no missing codes is to A/Ds, effective bits gives a useful insight into a Sampling A/D's transfer function as a function of frequency.

In correlating effective bits to signal-to-noise ratio, a correction factor for the formula is needed when the sinusoid input is less than full scale:

$$\text{correction factor} = \frac{+20\log \dfrac{\text{Full scale amplitude}}{\text{actual input amplitude}}}{6.02}$$

Numerical methods such as least squared minimization techniques can be used in comparing the ideal sinewave to the digitized sinewave.

FREQUENCY DOMAIN TEST CONDITIONS

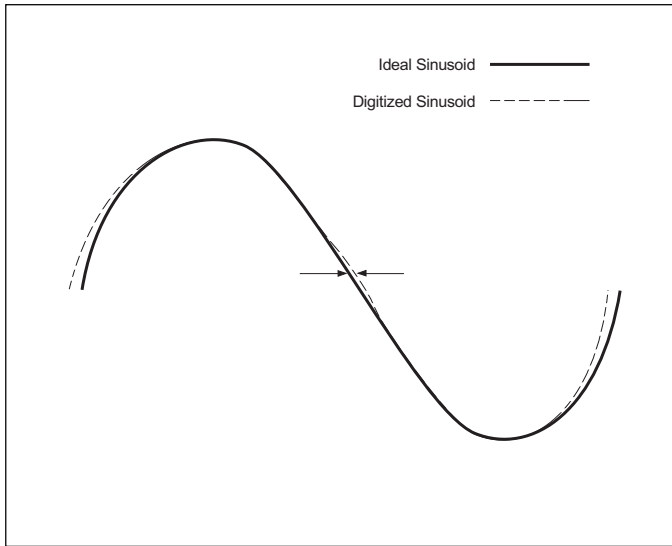FULL SCALE INPUT CONSIDERATIONS
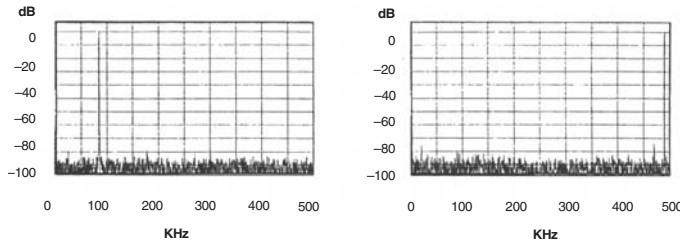
ADS–112 FFT@–4.44 dB OF FS		**ADS–112 FFT@–0.5 dB OF FS**	
2nd Harmonic	–88.1 dB	2nd Harmonic	–84.9 dB
Sinad	66.9 dB	Sinad	70.4 dB
Effective Bits	11.54 BITS	Effective Bits	11.41 BITS
DNL	±0.48 LSB	DNL	±0.55 LSB
Fs	1 MHz	Fs	1 MHz
Fin	490 kHz	Fin	490 kHz

$$SNR = 6.02n + 1.76dB - 20 \log \frac{\text{Full Scale Amplitude}}{\text{Actual Input}}$$

Effective Bits

As for many of the frequency domain tests, in performing the sine wave curve fit, it is important to assure that the input is not harmonically related to the sample frequency, i.e. the input frequency is not a submultiple of the sample frequency. Certain codes would increase in occurence and SNR might improve as the harmonics alias back onto the fundamental.

The number of data points, Dm, should also be large giving good frequency resolution. The actual frequency can then be readily determined.

Effective Bits uses numerical methods to compare how much a digitized sinewave represents an ideal mathematical model.

D/A >>>>> Monotonicity

A/D >>>>> No missing codes

ADS >>>>>Effective Bits

Ideal Sinewave: $A\sin(2\pi ft + \phi) + DC$

Where: A,f,ϕ, and DC are calculated to "best fit" the A/D data

   A = Amplitude

   f = Frequency

   Itl = Phase

   DC = DC Offset

The RMS error between an ideal sinewave and the best-fit sinewave is:

$$Error_{rms} = \sqrt{\sum_{m=1}^{N} [Dm - A\sin(2\pi ftm + \phi) - DC]^2}$$

Where: E = Calculated rms error

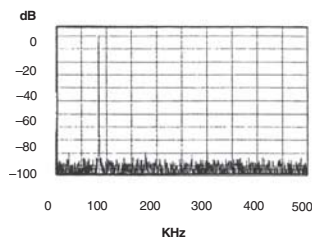   N = Data record length

        Dm = The data

Taking the partial derivative of Erms with respect to each of the four parameters yields:

$$\text{Effective bits} = n - \log_2 \frac{E_{rms}}{q/\sqrt{12}}$$
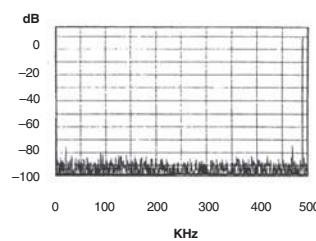
Where q = LSB size

FREQUENCY DOMAIN TEST CONDITIONS

FREQUENCY CONSIDERATIONS

ADS–112 FFT@90 KHz

2nd Harmonic	–88.2 dB
Sinad	71.1 dB
Effective Bits	11.58 BITS
DNL	±0.51 LSB
Input Signal	–0.5dB of FS
Fs	1 MHz

ADS–112 FFT@–0.5 dB OF FS

2nd Harmonic	–84.9 dB
Sinad	70.4 dB
Effective Bits	11.41 BITS
DNL	±0.55 LSB
Input Signal	–0.5dB of FS
Fs	1 MHz

Two-Tone Intermodulation Distortion

Total IMD = $20 \log \left(\dfrac{\text{RMS of sum \& difference of distortion products}}{\text{RMS value of fundamental}} \right)$

$$= 20 \log \sqrt{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [(IMD(m\omega, \pm n\omega_2)]^2}$$

Total IMD = $20 \log \sqrt{[_{10}(IMD@ 2F_1 + F_2/20)]^2 + [_{10}(IMD@ 2F_1 + F_2/20)]^2 +}$

$\sqrt{[_{10}(IMD@ 2F_1 + F_2/20)]^2 + [_{10}(IMD@ 2F_1 + F_2/20)]^2 + [_{10}(IMD@ 2F_1 + F_2/20)]^2}$

$\sqrt{[+ [_{10}(IMD@ 2F_1 + F_2/20)]^2 + ...}$
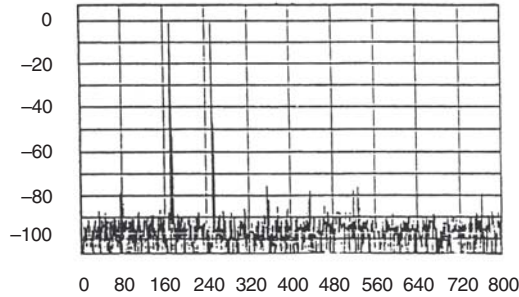
Correlative Effective Bits with SNR (& Distortion):

Effective Bits = $\dfrac{SNR - 1.76 + \left[20 \, Log \dfrac{\text{Full scale amplitude}}{\text{Actual input amplitude}} \right]}{6.02}$

Input level correction factor

$SNR = 6.02n + 1.76dB - 20 \, Log \left[\dfrac{\text{Full scale Amplitude}}{\text{Actual input amplitude}} \right]$

Shown are two equal amplitude sinusoids, Sin(2p F1 t) and Sin(2p F2 t) of different frequencies F1, F2.

Harmonics still appear at nF1 and nF2 where n is an integer

Now also see components at mF + nF and mF – nF where m,n are any integers

2nd Order terms: $(F_1 + F_2)$, (176 KHz + 256 KHz = 432 KHz)

$(F_1 - F_2)$, (176 KHz – 256 KHz = –80KHz)

3rd Order terms: $(2F_1 + F)$, (2 x 176 KHz + 256 KHz 608 KHz)

$(2F_1 - F_2)$, (2 x 176 KHz – 256 KHz = 96 KHz)

$(F_1 + 2F_2)$, (176 KHz + 2 x 256 KHz = 688 KHz)

$(F_1 - 2F_2)$, (176 KHz – 2 x 256 KHz = -336 KHz)

Noise Power Ratio (NPR)

Peak to Peak Noise

K	Out of Range	In Range
2	32%	68%
4	5%	95%
6	.3%	99.7%
8	.005%	99.995%
10	.0003%	99.9997%

K=2
K=4
K=6
K=8
K=10

Loading Factor (K) Versus (σ)

Input Bandwidth

A_{dB}

Large Signal Bandwidth

Full Power
Bandwidth

Small Signal Bandwidth

f_c

Large Signal Bandwidth:
Frequency where the maximum sinusoidal input signal has decreased by 3dB as derived from the digital output data.
Full Power Bandwidth:
Maximum frequency full scale sine wave digitized without spurious or missing codes.
Small Signal Bandwidth:
Bandwidth where the amplitude is 1/10 the value (–20dB) of the maximum input amplitude.

Two-Tone Intermodulation Distortion

Typically used in communication applications such as when multiple frequencies are multiplexed onto a single carrier, two-tone intermodulation distortion is the output distortion resulting from one sinusoidal input signal's interaction with another signal at a different frequency (tone).

As in harmonic distortion, nonlinearities in the A/D transfer in the frequency domain as intermodulation distortion, occurring for two-tone inputs at the sum and difference frequencies.

An FFT is shown with two input frequencies (tones) applied. The input amplitudes should combine to produce a resultant signal amplitude of –0.5 dB down from full scale. Utilizing individual signals that are less than –6 dB down from full scale will prevent clipping of the input signals. Also, these input signals should not be closer than the frequency resolution of the FFT.

Normally, two input signals close to the upper end of the bandwidth (1/2 the sampling rate) are used as test signals (lower frequency tones shown here to easily demonstrate the IMD). This results in the 2nd order terms being spaced far apart from the original signals while there are third order terms which are spaced close to the input signals. The third IMD terms which are close to the input signals are difficult to filter.

Noise Power Ratio

Noise Power Ratio (NPR) is a specification developed for Frequency Division Multiplexed Communication Equipment. In a multichannel environment, the signals in the off channels can create thermal and intermodulation effects on the channel of interest.

Practical NPR testing simulates the random noise which would occur in the multichannel environment through the use of a noise generator. A narrow-band notch filter is switched in and out and a noise receiver utilized in determining the ratios. FFT base(NPR testing usually demands excessive data records to obtain the relevant frequency resolution for the band of interest.

For an ideal A/D:

$$NPR = \frac{\text{Full scale of the A/D}}{K(\text{Quantization Noise})} \qquad \frac{Q2^n}{KQ\sqrt{12}} \qquad \frac{2^n}{K\sqrt{12}}$$

Where:

Q = Quantization Level

($Q\sqrt{12}$ is Quantization Noise)

n = # of bits

$K = \text{Loading Factor} \quad \dfrac{(\text{Full Scale})}{\text{RMS Noise Level}}$

$$NPR_{dB} = 6.02n + 20\log(\sqrt{3}/K) \text{ dB}$$
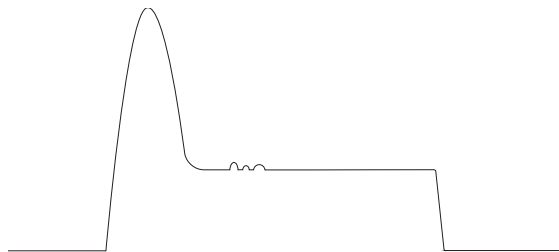
TRANSIENT RESPONSE & OVERLOAD RECOVERY TIME

Transient Response >>>>> Acquisition Time

Overload Recovery Time:

Time after the input returns to the A/D converter's normal operating input range until the time that the A/D can make a proper conversion.

The overvoltage condition needs to be specified. ± 20%?

± 2X INPUT?

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Testing is performed at low, medium and high frequencies as various errors from nonlinearity, intermodulation or crossover distortion, and phase distortion may be more prevalent at certain frequencies.

The Noise Power Ratio (NPR) calculation requires a loading factor, K, be used to prevent clipping and its associated distortion.   Here, the peak-to-peak noise is graphed as, function of sigma (a = rms noise level).

For a 12-bit accurate system, choosing a value of 8 for K will assure that only a small amount of the noise (1 LSB = 0.024% FSR) i: outside the full scale range of the converter

Input Bandwidth

A/D converters should have wide-input bandwidths for handling transients or pulse type analog inputs. Sinusoids digitized at Nyquist undergo full scale steps from one conversion to the next. However the step function is not able to track the input signal between conversions and presents a more stringent test.

A step function, such as created when a multiplexer switches between multiple channels or perhaps full scale steps between pixels (white/dark edges) in an imaging application, may also undergo full-scale   steps from one conversion to the next

Wide input bandwidths also prevent phase shifts from occurring in such applications as I *&* Q channels in radar. Additionally, fast recovery times to input overvoltage conditions which are not clamped are achieved when wide input bandwidths are present.

Transient Response

Overload Recovery Time

Acquisition time, the time needed for a sample-hold to acquire a full scale step signal is sometimes referred to as transient response. Full scale steps, pulses or transients can be a more difficult test for an A/D than a sinusoidal type input. A wide input bandwidth and fast acquisition time (transient response) assures the A/D can handle these transients.

ADS-130

Key specifications for determining how an A/D converter will handle sinusoidal or pulse/step applications are shown. Note the minimum-maximum specifications over temperature and the frequency< domain specifications as a function of the full scale input (–0.5 dB) and frequency.

Frequency Domain Test Conditions

FFT Size/Accuracy Considerations

The size of the FFT (number of data points) utilized determines the frequency resolution and the accuracy of the FFT. The table shows that taking more samples improves the accuracy of the FFT, however note that the square root relationship prevents a dramatic accuracy gain. The 4096 point FF yields a 0.06% accuracy which is more suitable for 12-bit A/Ds.

$$\text{FFT Accuracy} = \frac{\pm 4}{n\sqrt{N}} \text{ dB}$$

Where: n = # of bits resolution

N = number of data points

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Functional Specifications

Dynamic Performance	Min	Typ	Max	Units
Slew Rate	175	200	—	V/µSec
Aperture Delay Time	—	5	7	nSec
Aperture Uncertainty	—	5	7	psec
S/H Acquisition Time				
to 0.1% FS (2.5V step)				
+25°C	—	30	50	nSec
0 to +70°C	—	30	50	nSec
–55 to +125°C	—	30	50	nSec
Conversion Rate				
(Changing Inputs), +25°C	10	—	—	MHz
0 to +70°C	10	—	—	MHz
–55 to +125°C	10	—	—	MHz
Total Harm. Distortion (–0.5dB)				
DC to 500KHz	–68	–70	—	FS,–dB
500KHz to 2.5MHz	–65	–67	—	FS,–dB
2.5MHz to5MHz	–65	–67	—	FS,–dB
Signal-to-Noise Ratio				
(w/o distortion, –0.5dB)				
DC to 500KHz	–67	–70	—	FS,–dB
500KHz to 2.5MHz	–65	–69	—	FS,–dB
2.5MHz to5MHz	–65	–69	—	FS,–dB
Signal-to-Noise Ratio and distortion, (–0.5dB)				
DC to 500KHz	–65	–66	—	FS,–dB
500KHz to 2.5MHz	–63	–65	—	FS,–dB
2.5MHz to5MHz	–63	–65	—	FS,–dB
Spurious Free Dynamic Range				
DC to 500KHz (–0.5dB)①	–69	–70	—	FS,–dB
500KHz to 2.5MHz	–66	–67	—	FS,–dB
2.5MHz to5MHz	–66	–67	—	FS,–dB
Effective Bits				
DC to 500KHz	10.6	11.0	—	Bits
500KHz to 2.5MHz	10.2	10.5	—	Bits
2.5MHz to5MHz	10.0	10.2	—	Bits
Two Tone Intermodulation Distortion (fIN = 2.2MHz, 2.3MHz, Fs=8MHz)	–72	–75	—	dB
Input Bandwidth				
Small Signal (–20dB)	50	65	—	MHz
Large Signal (–3dB)	30	40	—	MHz
Feedthrough Rejection				
2.5V step	–62	–66	—	dB
Overvoltage Recovery, ±2.5V	—	50	100	nSec

Coherent and Noncoherent Sampling

Coherent Sampling:

An integer number of sinewave cycles are used in the data record for the FFT.

Frequency content of the input signal has to be known.

Provides the largest dynamic range.

Eliminates "leakage" and the need for "windowing".

Noncoherent Sampling:

A non-integer multiple of the sinewave cycle is in the data Record

Needed when the frequency content of the input signal is unknown

A "window" function to time-weight the data is required
At the heart of this mathematical exercise in transforming time domain data to the frequency domain is the goal that complete cycles of sinusoidal waveforms are being analyzed. Coherent sampling assures that complete cycles are used.
The Fourier Transform expects to work on continuous data from $-\yen$ to $+\yen$. In performing an FFT, the infinite amount of data has been truncated to contain a finite number of samples.

Noncoherent sampling utilizes windowing functions to approximate this ideal. This is accomplished by having the data at the ends (partial cycle) go to zero. Careful selection of the window function is required to minimize the fundamental frequency leaking into other frequency "bins."

Coherent Sampling

The Analog Input sinewave frequency and the A/D sampling rate need to observe the following relationship for coherent sampling:

$$f_{in} = \frac{nFs}{N}$$

Where:

f_{in} =   Input Sinewave Frequency

Fs =   Sampling Rate

n =   Prime Number (1,3,5,7,11 ETC)

N =   Number of Samples Taken

Note: The input frequency and the sampling rate need to be frequency locked.
Coherent sampling requires the input frequency to be frequency locked to the A/D sampling rate. Assuring complete cycles eliminates the need for windowing functions to force the data at the boundaries to zero.
Coherent sampling becomes important with higher resolution A/Ds as the window functions reach their limit. The 4-term Blackman-Harris window's sidelobe is -92 dB down and begins to be a factor in 14 and 16-bit A/D testing.

Noncoherent sampling

FFT window aspects

Frequency Range/Resolution

Spectral Leakage

3dB Bandwidth

Picket-fence effect/Scallop loss

Equivalent Noise Bandwidth

Worst case processing loss

Noncoherent sampling is required when the input frequency is unknown and is therefore more representative of an actual A/D application. The windows utilized in non-coherent sampling attenuated the sidelobe errors while spreading out the main-lobe response.
A number of characteristics of the window chosen and FFT become important depending on the application.

Frequency Range/Resolution of the FFT

$$\text{Frequency Range of the FFT} = Fr = \frac{\text{Sampling Rate}}{2}$$

$$\text{Minimum Frequency Resolution of the FFT (\# of Frequency Bins)} = \Delta F = \frac{\frac{\text{Sampling Rate(Fs)}}{\text{Record}}}{\text{Length (N)}}$$

$$\text{Frequency Resolution} = \Delta F = \frac{\text{(ENBW) Sampling Rate}}{\text{Record Length}}$$

Where ENBW = Equivalent Noise Bandwidth

Per Nyquist, the frequency range, Fr, is the Sampling rate divided by two. The frequency resolution, Fs/N, is the minimum FFT bin width. Depending on the particular window utilized, the bin width increases by the Equivalent Noise Bandwidth (ENBW).

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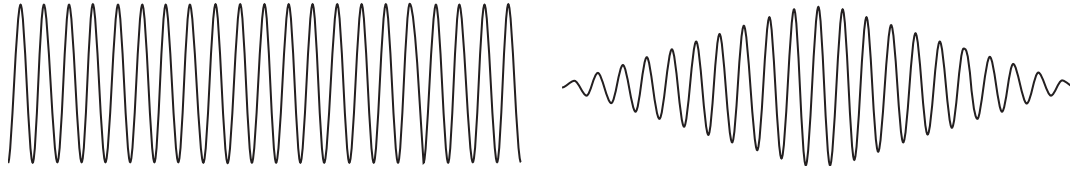
SPECTRAL LEAKAGE

Spectral leakage is caused by finite data records not representing complete cycles. Discontinuity at the boundaries occur when complete cycles are not represented

Window functions weight the data heavily in the center of the data record and bring the signal to zero at the boundaries

Spectral leakage occurs when energy from one frequency spreads into adjacent ones. This occurs when a fraction of a cycle exists in the waveform that is subjected to the FFT. Window functions are used to reduce the spectral leakage.

SPECTRAL LEAKAGE

A_K = Magnitude of Sine
B_K = Magnitude of Cosine
W_m = Weighting function
D_m = Amplitude of the Data Point
 N = Number of Data Points
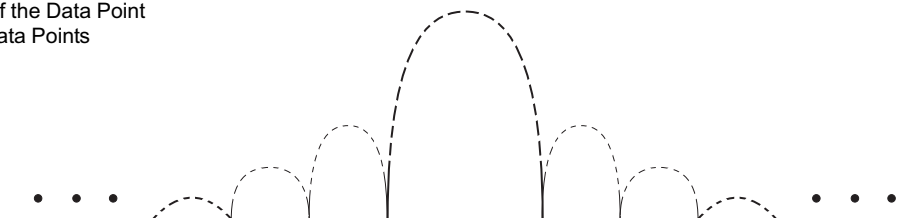
The windowing of the data is similar to multiplying the signal by a rectangular window. Multiplication in the time domain is analogous to convolution in the frequency domain.

The fourier transform of a rectangular window is the popular (sinX)/X or sinc function. Convolving the spectrum of data, x(nΔt), with this function smears the calculated full spectrum of x(t). The spectral amplitudes of x(f) will leak through the side lobes of the sinc function, spreading energy from one frequency into adjacent frequencies.

The various window functions modify the main lobe and side lobes of the basic sinc function shown for a rectangular window.

3.0 dB Bandwidth (in bins)

3.0dB Dip

Non-resolvable signal peak   Resolvable signal peak
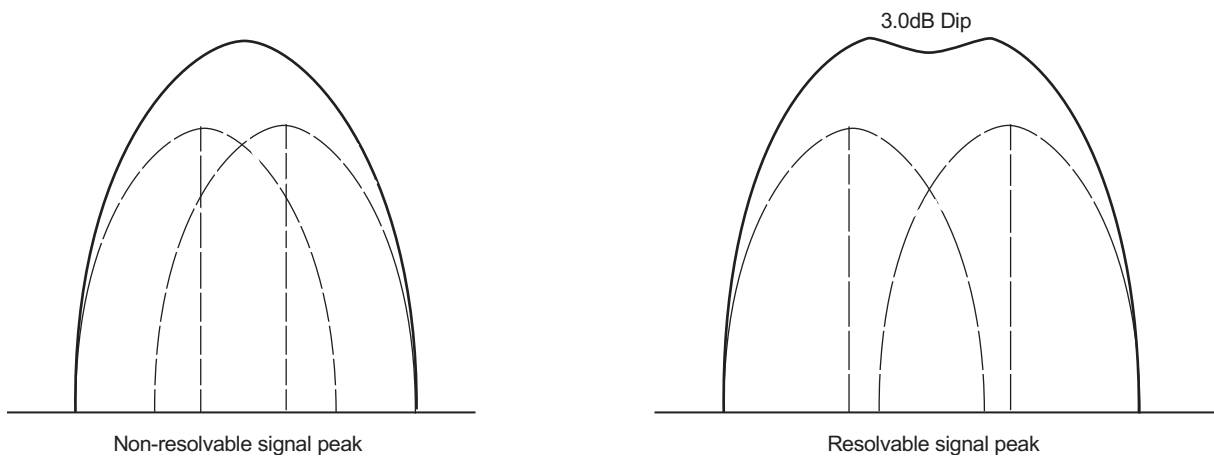
The 3.0 dB bandwidth (in frequency bins) is defined as the minimum bandwidth where two closely spaced signals can still be resolved. Two signals separated by less than their 3.0 dB bandwidth are erroneously resolved into a single spectrum component.

The ideal 3.0 dB bandwidth is equal to one FFT bin width, requiring a windowing function with characteristics of an ideal bandbass filter (0 dB in the passband, steep attenuation outside).
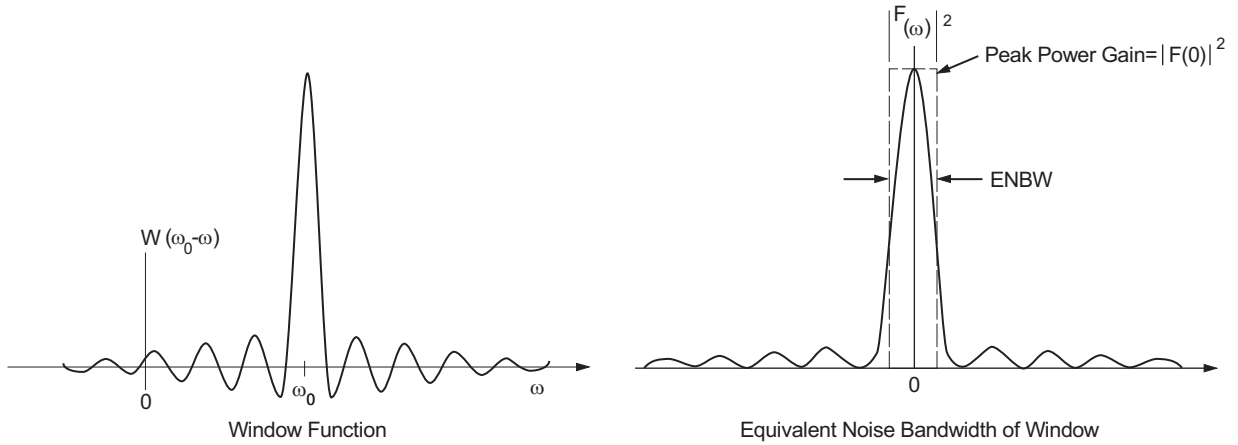
PICKET FENCE EFFECT (SCALLOP LOSS)

Loss in gain for a frequency midway between two bin frequencies
Scallop loss varies depending on window chosen

Amplitude of the kth spectral line = $\sqrt{A^2_K + B^2_K}$

Where: $A_K = \dfrac{1}{N} \sum\limits_{m=0}^{N-1} W_m D_m Cos \left[\dfrac{2\pi k(m-1)}{N} \right]$

$B_K = \dfrac{1}{N} \sum\limits_{m=0}^{N-1} W_m D_m Sin \left[\dfrac{2pk(m-1)}{N} \right]$

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EQUIVALENT NOISE BANDWIDTH (ENBW)

Window Function

Equivalent Noise Bandwidth of Window

Determining the spectral amplitude of a signal is affected by the noise in the bandwidth of the window
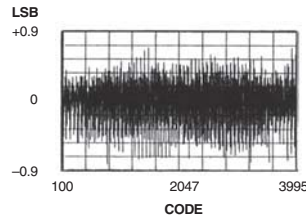
Narrow bandwidth filters are used to minimize the noise

ENBW is the width of a rectangular filter with the same peak power gain which would acquire the same noise power as the windowing function utilized.

HISTOGRAMS (CODE DENSITY TEST)
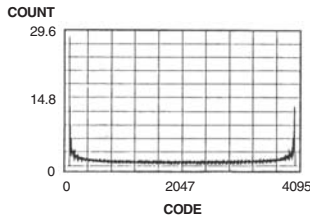
ADS-131 HISTOGRAM

@2.45 MHz Fin

ADS-131 DIFFERENTIAL

NONLINEARITY@2.45 MHz Fin

Histogram used in testing differential nonlinearity
Sinewave Inputs versus DC Ramps allow testing as a function of frequency

$$\text{Probability function for a sinewave} = P(V) = \frac{1}{\pi \sqrt{A^2 - V^2}}$$

Where: A = Peak Amplitude of Sinewave
        P(V) = Probability of Occurence at Voltage (V)

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HISTOGRAM ACCURACY

Number of samples (Nt) required for a particular precision in LSB's (β)
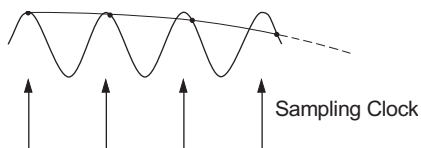and confidence (C) for an n-bit converter is:

$$Nt = \frac{(Z_\infty)^2 \, \pi \, 2^{n-1}}{\beta^2}$$

Where: Z_∞ = # of standard deviations
for any chosen a

$$\infty = \frac{100 - C}{200}$$

| C%	99	98	97	96	95	94	93	92	91	90
∞	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
Z_∞	−2.58	−2.33	−2.17	−2.06	−1.96	−1.88	−1.82	−1.75	−1.7	−1.65

BEAT FREQUENCY TEST

Sampling Clock

Beat frequency testing is a figure of merit test giving quick insight into an A/D's gross dynamic errors
Differential nonlinearities and missing codes appear as distortion or discontinuity in the low frequency sinewave
The beat frequency is set to change by one LSB from one conversion to the next

$$\Delta f = \frac{Fs}{2^n \pi}$$

Where: Df = Beat frequency
Fs = Sampling rate
n = #of bits

ENVELOPE TEST
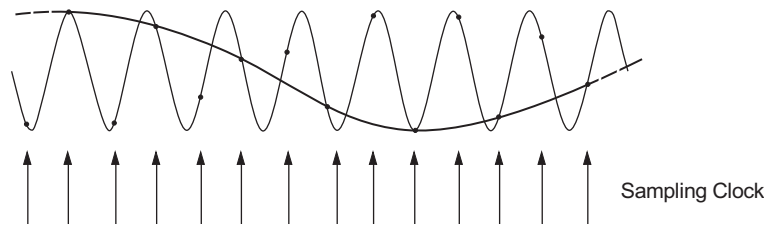
Similar to beat frequency test with the offset frequency being at two samples per cycle
The A/D is required to slew effectively from peak to peak on the sinewave from one conversion to the next
A digital divider is used to send every other sample to the D/A, allowing a low frequency beat frequency to be observed.

Accuracy of FFT

Number of Data Points:	**256**		**4096**	
Resolution				
12-BIT	0.021dB	0.24%	0.005dB	0.06%
14-BIT	0.018dB	0.21%	0.0045dB	0.051%
16-BIT	0.016dB	0.18%	0.0039dB	0.045%

Picket Fence effect (Scallop Loss)

The picket-fence effect or scallop loss (dB) can be significant for multi-tone signals. The FFT output points (frequency bins) occur at the sample frequency divided by the number of FFT points. The deviation in gain from 0 dB within +1/2 frequency bi is the scallop loss.

This scallop loss can vary from 3.92 for the rectangular window to 0.83 for the 4-term Blackman-Harris window.

Equivalent Noise Bandwidth (ENBW)

A window function acts like a filter, weighting the inputs for the final calculation over its entire bandwidth. To detect low-level signals, windows with narrow bandwidths are used to minimize the noise.

The equivalent noise bandwidth (in frequency bins) can vary from 1.0 for the rectangular window to 2.0 for the 4 term Blackman-Harris window.

Histograms (Code Density Test)

The histogram plot of the 12-bit, 5 MHz ADS-131 sampling A/D shows the possible output codes along the X axis and the frequency of occurrence along the Y axis for a sinewave. This is then compared to the probability density function of an ideal sinusoid in determining the differential nonlinearity of the A/D.

Sinusoids allow testing differential nonlinearity as a function of the input frequency (non-coherent sampling). More occurrences are seen at the peak of the sinewave versus at the zero crossing where maximum slew rate of the sinusoid occurs. Ramp testing as an input for a histogram is usually used for slow changing signals only. Discontinuities at the peak would require infinite bandwidth for no distortion.

Histogram Acuuracy

The differential nonlinearity for a 12-bit converter with a 99% confidence to 0.1 LSB accuracy would require 4.3 million samples. Dividing the 4.3 million samples by 4096 codes (2 where n =12) is about 1000 records. Using 100 records yields 1/4 LSB accuracy.

Histogram testing is not popular for integral nonlinearity. Integral nonlinearity would require more samples and associated longer testing time. Drifts in the input signal's amplitude or offset or the A/D's offset or gain would produce erroneous results. An alternative method would be to use an FFT where nonlinearity appears as harmonics.

Beat Frequency Test

Setting the full scale input sinewave frequency equal to the sampling rate of the A/D would ideally give the same output code for each conversion. Offsetting the input sinewave frequency such that from one conversion to the next a maximum of one LSB change occurs results in a beat frequency, *Af.*

This slow beat frequency gives a quick, visual demonstration of the A/D's performance utilizing computer graphics or reconstruction D/As. The D/A's update rate should be frequency locked with the A/D sampling rate.

Envelope Test

The envelope test is more demanding than a beat frequency test as it requires the A/D to slew from one extreme of the full scale range to another on successive conversions.

Worst Case Processing Loss

Combines Scallop loss with Processing loss which occurs at the data record boundaries.

The worst case processing loss reduces the signal-to-noise ratio due to windowing and the worst case

Mainly an issue for multi-tone testing.

The worst case processing loss varies from 3.0 to 4.3 dB for various windowing functions. This loss becomes important when detecting low level signals in the presence of many other frequencies where contribution to a spectral component may be shared by different frequency bins.

Common FFT windowing functions

Four term Blackman-Harris

Flatness of the main lobe-Accurate peak and power measurements

Hanning

Improved frequency resolution over rectangular and triangular windows

Hamming

Optimized to lower the 1st side lobe

Detection of close frequency lines improved

Rectangular

Transient signals

Highest sidelobes limit ability to:

(1) Resolve two close spectral components

(2) Detect low amplitude frequencies (masked by leakage)

Triangular

Cannot detect a weak spectral ilne in the presence of strong signals

FFT window characteristics

A variety of window functions exist with five of the more common listed here. Window selection may depend on ease of processing, single or multi-tone testing, presence of a small signal in the near vicinity of a large signal, etc..

Some of the major characteristics are shown for some of the common FFT windows. Note the -92 dB side lobe for the 4-term Blackman Harris window used in testing precision A/Ds

Low-pass Filters for Signal Generators

Harmonics Observed on the FFT Plot may be due to:

Signal Generators

Resolution/Frequency dependant

Filter used to remove the signal generator harmonics

Drive/Impedance matching when passive filter used

Test Fixturing

The A/D Converter

Passive versus Active filters

Passive filters

Utilize resistors, capacitors and inductors

Lower noise than active type especially at high frequencies

Less harmonic distortion

No power supply required

Impedance matching issues

Large size

Stable versus frequency

	Highest Sidelobe (dB)	3.0 dB Bandwidth (Bins)	Scallop Loss (dB)	ENBW (Bins)	Worst Case Process Loss (dB)
BLACKMAN-HARRIS	-92	1.90	0.83	2.0	3.85
HANNING (COS2X)	-32	1.44	1.42	1.5	3.18
HAMMING	-43	1.3 1	.78	1.36	3.10
RECTANGULAR	-13	0.89	3.92	1.00	3.92
TRIANGULAR	-27	1.28	1.82	1.33	3.07

Active Filters

Utilizes op amps,resistors and capacitors

Bulky and expensive inductors eliminated

Easy to control gain

Tunable

Small size

Low weight

Minimal shielding problems

High input impedance

Low output impedance

Careful analysis of the sources of the harmonics appearing on an FFT plot of an A/D would go beyond the A/D itself. The signal
generator, anti-alias filtering, and test fixturing are all potential causes of harmonics.

A comparison of the virtues of passive versus active filters are shown. As the resolution and frequency increase, the passive filters become the filters of choice.

Common Anti-alias Filter Types

Butterworth

Flatest response near DC, moderately fast roll-off, attenuation rate + 6dB/octave, stable phase shift, Fc occurs at –3dB, constant amplitude emphasis versus time delay or phase response. Overshoots on step response.

Chebyshev

Rapid attenuation above the cutoff frequency with some passband ripple,FC occurs when the attenuation exceeds the specified ripple. Has a squarer amplitude response than the Butterworth but less desirable phase and time delay.

Cauer (elliptical)

Surpasses other designs for critical amplitude applications, very sharp rolloff rate with some ripple, squarest possible amplitude response with poor phase and transient response

Bessel

Optimized flat phase response over wide input frequency, moderate attenuation rate, FC occurs where phase shift is !/2 the max phase shift, amplitude not as flat as Butterworth, rolloff is slow, gain rolloff could modify amplitude, important in pulse transmissions as avoids overshoot/undershoot

Some of the common anti-alias filter types are listed. For frequency analysis of high resolution, high-speed A/D converters, the Cauer (Elliptical) filter finds broad usage.

Final Summary

Seminar Objectives:

Upon completion of this seminar you will be able to understand the various A/D converter frequency domain specifications.

Instructor Biography:

Bob Leonard is the Product Marketing Manager for Component Products at DATEL. Bob received his BSEE degree from Northeastern University and has been with DATEL since 1974. In his spare time, Bob enjoys reading and various sports including golf, tennis, basketball, softball, soccer and running.

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